10. Spectral Lines I

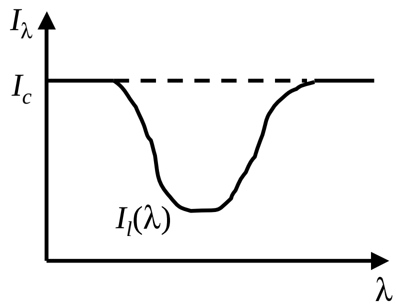
- Rutten: 9.1, 3.3
- Line strength, equivalent width
- Line profile
- Broadening
- Voigt profile

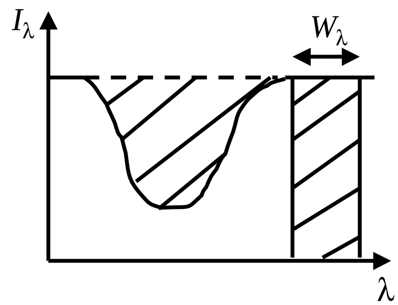
The Line Spectrum

A spectral line is described by its profile. The line depression, D_λ, compares the intensity in the line with the nearby continuum. The equivalent width is the integral of D_λ and is the same width as a rectangular piece of spectrum that blocks the emergent intensity:

$$\boxed{D_\lambda = \frac{I_c - I_\lambda^l}{I_c}}$$

$$\boxed{W_\lambda = \int_{\text{line}} D_\lambda \, \mathrm{d}\lambda = \int_{\text{line}} \frac{I_c - I_\lambda^l}{I_c} \, \mathrm{d}\lambda}$$



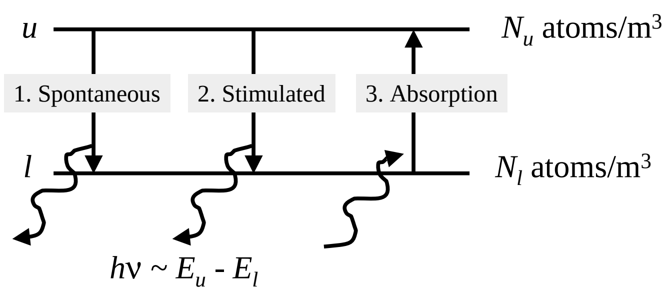


Observationally: Line profiles for weak lines difficult to determine accurately due to distortion caused by finite width of spectrograph slit. Total energy subtracted from continuum is not affected, this measures the integrated line strength or equivalent width.

Theoretically: Line intensity can be calculated from ERT if α_ν is known. These depend on atomic transition coefficients, level occupation numbers and the intrinsic line profile for the relevant transition. The profile gives the probability of a photon with ν being absorbed or emitted. This depends on the atmospheric temperature and pressure at the particular depth considered. The problem can be broken into two parts: investigation of intrinsic line profile
solution of ERT for the line

An important assumption is that line and continuum can be solved separately. If line abs/emis is strong this is not valid and T-structure depends on line transfer and line/cont must be solved simultaneously.

Intrinsic Line Profile



Referring back to L3, processes contributing to the line profile are:

1. Spontaneous emission:	Spontaneous de-excitation: A_{ul}
2. Stimulated emission:	Induced de-excitation B_{ul}
3. Absorption:	Radiative excitation: B_{lu}

Number of spontaneous transitions /m^3 /sec = $N_u A_{ul}$
Number of stimulated transitions /m^3 /sec = $N_u B_{lu} J_{\nu 0}$
Number of spontaneous transitions /m^3 /sec = $N_l B_{ul} J_{\nu 0}$

Line Broadening Mechanisms

Recall that for complete redistribution the spontaneous emission, stimulated emission, and absorption line profile shapes are the same. Lines are not sharp due to some or all of the following broadening mechanisms:

- Natural or radiation damping: lifetimes of excited states
- Collisional: collisions or perturbations by other particles
- Doppler: thermal motions
- Rotational: stellar rotation
- Turbulent: mass motions in atmosphere
- Zeeman splitting: magnetic effects

1. Natural Broadening: From L3, the mean lifetime of particles in state u is $\Delta t = 1/A_{ul}$ seconds. The corresponding spread in energy is (Heisenberg) $\Delta E = h/(2\,\pi\,\Delta t)$ or $\Delta\nu = \gamma^{\mathrm{rad}}/2\,\pi$ with $\gamma^{\mathrm{rad}} = 1/\Delta t$ the *radiative damping constant*. This "natural" broadening yields the Lorentz profile:

$$\boxed{\psi(\nu-\nu_0) = \frac{\gamma^{\mathrm{rad}}/4\pi^2}{(\nu-\nu_0)^2 + (\gamma^{\mathrm{rad}}/4\pi)^2}}$$

γ = "damping" because Lorentzian comes from solution of atom as a harmonic oscillator: $\boxed{m\,\ddot{x} + \gamma\,\dot{x} + \omega^2 x = 0}$

In real atoms the lower level, l, may have a finite lifetime and there may be multiple transitions from each level. The transition probs add up as: $\boxed{\gamma_u^{\mathrm{rad}} = \sum_{l<u} A_{ul}}$ The total damping width for a line is given by:

$$\boxed{\gamma^{\mathrm{rad}} = \gamma_l^{\mathrm{rad}} + \gamma_u^{\mathrm{rad}} = \sum_{i<l} A_{li} + \sum_{i<u} A_{ui}}$$

2. Collisional Broadening: Excitations & de-excitations will occur through collisions between atoms. If t_c is the collision lifetime of an energy level, the energy spread is given by: $\boxed{t_c \, \Delta E \approx \hbar}$

Mean free path for collisions is $l = 1/n\sigma$, where s is collision cross section and n the atom number density. Hence, $t_c \sim l/v$ where v is mean speed of atom. Assuming Maxwellian: $v^2 = 3kT/m$ and hence

$$\boxed{t_c = \frac{1}{n\sigma}\sqrt{\frac{m}{3kT}}}$$

Spread in frequency of emitted photons is $\boxed{\Delta\nu \approx n\sigma\sqrt{\frac{3kT}{m}}}$

The effect on line profile is to add γ^{col} to Lorentzian.

Many more detailed calculations to describe interactions: Stark resonance, Van der Waals, etc

3. Doppler Broadening: Radiating particle moving along LOS gives Doppler shift (for $\xi << c$): $\boxed{\frac{\Delta\nu}{\nu} = -\frac{\Delta\lambda}{\lambda} = \frac{\xi}{c}}$

with ξ the component along LOS: +ve towards observer, blueshift. Observed ν of emitted photon is $\nu = \nu'(1+\xi/c)$. For absorbing atom moving towards observer, it extincts radiation it sees redshifted in its frame to $\nu' = \nu(1-\xi/c)$.

For thermal motion, the velocity distribution along LOS is Maxwellian:

$$\boxed{\frac{n(\xi)}{N}\,\mathrm{d}\xi = \frac{1}{\xi_0\sqrt{\pi}}\exp(-\xi^2/\xi_0^2)\,\mathrm{d}\xi}$$

where $\xi_0^2 = 2\,k\,T/\,m$.

Line extinction profile taking into account thermal motions is (Rutten 4.1.3):

$$\boxed{\sigma_\nu^l = \frac{\pi e^2}{m_e c} \frac{f}{\Delta\nu_D} \exp(-\Delta\nu^2 / \nu_D^2)}$$

with the Doppler width $\Delta\nu_D$:

$$\boxed{\Delta\nu_D \equiv \frac{\xi_0}{c}\nu_0 = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}}$$

In all of the above, we can relate $\Delta\lambda$ to $\Delta\nu$ by $\Delta\lambda \sim \lambda_0^2\,\Delta\nu / c$.

Other Broadening Mechanisms

Rotational Broadening: Stellar rotation, $\Delta\lambda \sim \lambda_0\, v/c \sin i$

Zeeman Splitting: Magnetic fields cause splitting (broadening).

Micro & Macro Turbulence: Mass motions in photosphere. Usually assumed to be Maxwellian in nature, so get:

$$\boxed{\Delta\nu_D \equiv \frac{\nu_0}{c}\sqrt{\frac{2kT}{m} + \xi^2_{\mathrm{micro}}}}$$

Macroturbulence is added by convolving emergent model line profile with a Gaussian velocity distribution ~ $\exp(-\xi^2/\xi^2_{\mathrm{macro}})$ BUT… these are VERY uncertain and are really just fudge factors to get model spectra to match observations.

The Voigt Profile

When collisional damping has Lorentz shape (impact approx), the total damping profile is Lorentzian with $\gamma = \gamma^{rad} + \gamma^{col}$. The total extinction coefficient is convolution of the damping and thermal profiles:

$$\begin{aligned}
\sigma_\nu^l &= \left[\frac{\pi e^2}{m_e c}\frac{f}{\Delta\nu_D}\exp(-\Delta\nu^2/\nu_D^2)\right] \otimes \left[\frac{\gamma/4\pi^2}{(\nu' - \nu_0)^2 + (\gamma/4\pi)^2}\right] \\
&= \frac{\pi e^2}{m_e c}\frac{f}{\Delta\nu_D}\int_{-\infty}^{+\infty}\frac{(\gamma/4\pi^2)\exp(-\Delta\nu^2/\nu_D^2)}{(\nu' - \nu_0)^2 + (\gamma/4\pi)^2}\,\mathrm{d}\nu \\
&= \frac{\pi e^2}{m_e c}\frac{f}{\Delta\nu_D} H(a,\nu)
\end{aligned}$$

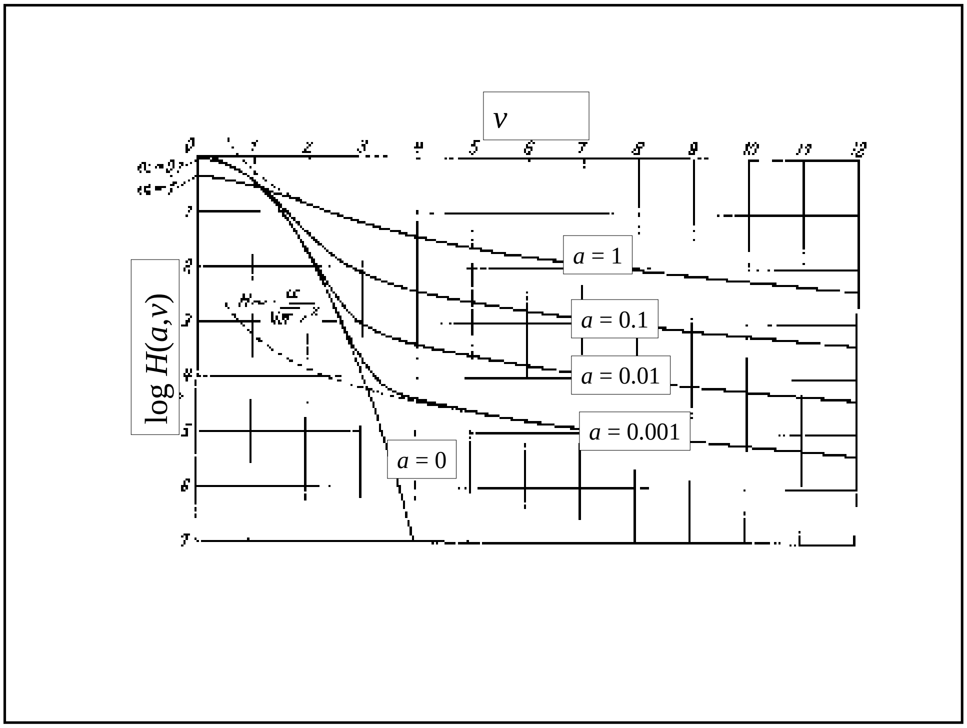
$$H(a,\nu) \equiv \frac{a}{\pi}\int_{-\infty}^{+\infty}\frac{\exp(-y^2)}{(v-y)^2 + a^2}\,\mathrm{d}y$$

$$y \equiv \frac{\xi}{\xi_0} = \frac{\xi}{c}\frac{\nu_0}{\Delta\nu_D}; \quad v \equiv \frac{\nu - \nu_0}{\Delta\nu_D}; \quad a \equiv \frac{\gamma}{4\pi\,\Delta\nu_D} = \frac{\lambda^2}{4\pi c}\frac{\gamma}{\Delta\lambda_D}$$

$H(a,v)$ is the Voigt Function. For a << 1 the rough approx is:

$$H(a,\nu) \approx \exp(-v^2) + \frac{a}{\sqrt{\pi}v^2}$$

So the profile is Gaussian near line centre, but has $1/\Delta\nu^2$ damping decay in the line wings. The Doppler core is quite wide. Only very strong lines have sufficient extinction far from line centre to show damping wings in emergent spectra.



v
log H(a,v)
a = 1
a = 0.1
a = 0.01
a = 0.001
a = 0